

# Propagation of the surface plasmon polaritons through gradient index and periodic structures

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## Abstract

We study the propagation of surface electromagnetic waves along the metallic surface covered by various layered dielectric structures. We show that strong radiative losses typical for the scattering of the surface wave can be considerably suppressed when single dielectric step is substituted by gradient index or periodic layered structure.

*Key words:* surface plasmon polaritons, gradient index structures, radiative losses, photonic crystal

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## 1. Introduction

It is well known that the life time of surface electromagnetic waves (surface plasmon polaritons - SPP) propagating along the metal-dielectric interface [1, 2, 3] is rather short. Even in the case of ideal lossless metal, the propagation of the SPP through any inhomogeneity is accompanied with radiative losses [4]. The simplest example is a transmission of the SPP through a single dielectric step, represented by the planar interface between two dielectrics which cover the metallic surface [5, 6]. As was shown recently [7, 8], SPP can radiate up to 40 per cent of its energy at the single interface.

The origin of radiative losses lies in the different spatial distribution of the electric and magnetic field in the direction perpendicular to the metallic surface. To balance the tangential components of  $\vec{E}$  and  $\vec{H}$ , the SPP radiates plane waves.

The radiative losses can be suppressed for SPP propagating at the interface between metamaterials. The losses are almost negligible for the TE polarized SPP propagating along the surface of negative permeability material. The physical reason for this is that the spatial distribution of the TE polarized SPP depends only weakly on the permittivity  $\epsilon_d$  of the dielectric [9]. In Ref. [10], anisotropic metamaterials were proposed to reduce radiative losses for the TM polarized SPP.

The presence of plane waves makes the quantitative analysis of the propagation of SPP mathematically difficult. Analytical calculation of the transmission and reflection coefficients [6] of the SPP through discontinu-

ities represents rather difficult task. Numerical methods [7, 8] are very useful for the quantitative analysis of the scattering. The problem is formulated for the scattering matrix of both SPP and radiative plane waves. With  $N_w$  plane waves considered, the size of the scattering matrix increases to  $2(1 + N_w) \times 2(1 + N_w)$ .

In this paper, we analyze the propagation of the SPP along the metallic surface covered by various dielectric materials. Of special interest is the propagation through layered structure when the metal is covered by parallel dielectric strips of various width and permittivities. We consider gradient index structures, in which the permittivities in two adjacent strips differs in small value  $\Delta\epsilon$ , and periodic structures composed of alternating dielectrics. We calculate the transmission and reflection coefficients for the SPP and show that radiative losses can be considerably reduced when the single interface between two dielectrics is substituted by the layered structure in which the permittivity smoothly changes.

The paper is organized as follows. In Section 2 we define the model and introduce the parameters of the SPP. Section 3 discuss the the method of calculation of the transmission parameters for SPP propagating through the layered structure. In Section 4 we show that radiative losses can be dramatically reduced when the single interface  $\epsilon_1/\epsilon_2$  is substituted by the gradient index structure [13], represented by  $N_w$  planar layers with linearly increasing dielectric permittivity. In Section 5 we discuss the propagation of the SPP through planar slab. Similarly to the case of the single interface, we find much higher transmission when the slab is modeled by

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the gradient structure (sect. 6). Propagation of the SPP through layered medium constructed by periodic alternation of two dielectrics  $\epsilon_1$  and  $\epsilon_2$  is analyzed in Section 7. We show that the propagation of SPP through such structure does not require an increase of radiative losses: radiation is the same, or even smaller, than that accompanying the propagation through single interface.

## 2. Definition of the model

Consider the metallic surface lying in the  $xy$  plane with metal filling the  $z < 0$  region. The metal is covered by different dielectrics. All interfaces between two neighboring dielectrics are parallel to the  $yz$  plane. The SPP propagates along the metallic surface. The direction of the propagation is perpendicular to dielectric interfaces. The electric and magnetic field of the SPP decreases exponentially in the  $z$  direction as  $\exp(-\kappa_d z)$  for  $z > 0$  (dielectric) and  $\exp(+\kappa_m z)$  for  $z < 0$  (metal). The analytical form of parameters  $\kappa_d$  and  $\kappa_m$  is [1]

$$\kappa_d^2 = -k_0^2 \frac{\epsilon_d^2}{\epsilon_d + \epsilon_m} \quad \text{and} \quad \kappa_m^2 = -k_0^2 \frac{\epsilon_m^2}{\epsilon_d + \epsilon_m}. \quad (1)$$

Here,  $k_0 = \omega/c$  and  $c$  is the light velocity,  $\epsilon_m$  is a permittivity of metal, represented by lossless Drude formula,  $\epsilon_m = 1 - \omega_p^2/\omega^2$  and  $\epsilon_d$  is dielectric constant of dielectric medium. The SPP propagates along the  $x$  direction. Then, its wave vector in the  $xy$  plane has only  $x$ -component,

$$k_x^2 = k_0^2 \frac{\epsilon_d \epsilon_m}{\epsilon_d + \epsilon_m}. \quad (2)$$

Equations (1,2) give complete frequency dependence of the wave vector of the SPP.

For the metal-dielectric interface only the TM polarized SPP can be excited [15]. The intensity of the magnetic field is

$$\vec{h} = \begin{cases} \mathcal{N}_0(0, 1, 0) \Phi(x, y, t) e^{-\kappa_d z} & z > 0, \\ \mathcal{N}_0(0, 1, 0) \Phi(x, y, t) e^{+\kappa_m z} & z < 0, \end{cases} \quad (3)$$

and the intensity of the electric field is

$$\vec{e} = \begin{cases} \mathcal{N}_0 \frac{z_0}{k_0} \Phi(x, y, t) (+i\kappa_d, 0, -k_x) e^{-\kappa_d z} / \epsilon_d & z > 0, \\ \mathcal{N}_0 \frac{z_0}{k_0} \Phi(x, y, t) (-i\kappa_m, 0, -k_x) e^{+\kappa_m z} / \epsilon_m & z < 0. \end{cases} \quad (4)$$

Here,  $\Phi(x, y, t) = e^{i(k_x x - \omega t)}$  and  $z_0 = \sqrt{\mu_0/\epsilon_0}$ . The constant  $\mathcal{N}_0$  normalizes the energy current in the  $x$  direction [11, 7].

Similar formulas can be derived for the plane waves scattered at the metallic interface. Explicit expression for the electric and magnetic fields is given elsewhere

[7, 8]. Here we only note that we will consider  $N_w$  plane waves with the same frequency  $\omega$  but different  $z$ -component of the wave vector  $k_z$ :

$$k_{z\alpha} = \frac{k_{z\max}}{N_w} \alpha, \quad \alpha = 1, 2, \dots, N_w. \quad (5)$$

Both  $N_w$  and  $k_{z\max}$  represent the parameters of the model. Other components of the plane wave vectors are  $k_y = 0$  and

$$k_{x\alpha} = \sqrt{k_0^2 \epsilon_d - k_{z\alpha}^2}. \quad (6)$$

Only waves with real  $k_{x\alpha}$  contribute to radiative losses.

In what follows we consider the frequency of the SPP

$$\omega = 0.23\omega_p, \quad (7)$$

which corresponds to the frequency of visible light, and neglect absorption. Then, the metallic permittivity is  $\epsilon_m = -17.9$ . Using Eq. 1, we express the wavelength  $\Lambda$  of the SPP:

$$\Lambda_{\epsilon_d} = \lambda_p \frac{\omega_p}{\omega} \sqrt{\frac{\epsilon_d + \epsilon_m}{\epsilon_d \epsilon_m}}. \quad (8)$$

where  $\lambda_p = 2\pi c/\omega_p \approx 94.2$  nm is the wavelength corresponding to the plasma frequency  $\omega_p/2\pi = 2 \times 10^{15}$  s<sup>-1</sup>.

## 3. Propagation of the SPP through planar interfaces

Scattering of SPP on the single interface is accompanied by the radiation losses [8]. Therefore, the scattering matrix  $T$  is of the size of  $2(N_w + 1) \times 2(N_w + 1)$ , where  $N$  is a number of plane waves, given by the  $z$ -component of the wave vector. The amplitudes of scattered waves are related by the scattering matrix

$$\begin{pmatrix} B \\ \bar{A} \end{pmatrix} = \begin{pmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{21} & \mathbf{S}_{22} \end{pmatrix} \begin{pmatrix} \bar{B} \\ A \end{pmatrix}. \quad (9)$$

Here,  $A_i$  and  $B_i$  are the amplitudes of waves on the left and right side of the interface. Amplitudes  $\bar{A}_i$  and  $\bar{B}_i$  belong to the plane waves propagating in the opposite direction. We have  $N_w + 1$  waves, ( $i = 0 \dots N_w$ ) with  $A_0$  and  $B_0$  representing the amplitudes of the SPP.

The scattering matrix expresses the waves propagating away from the interface ( $B, \bar{A}$ ) through the amplitudes of the incident waves ( $A, \bar{B}$ ). Explicit form of scattering matrix is given in [7, 8]. The transmission and the reflection coefficients of the SPP incident from medium 1 are given by

$$T_{1 \rightarrow 2} = |\mathbf{S}_{12}(00)|^2, \quad R_{1 \rightarrow 1} = |\mathbf{S}_{22}(00)|^2, \quad (10)$$

and from medium 2

$$T_{2 \rightarrow 1} = |\mathbf{S}_{21}(00)|^2, \quad R_{2 \rightarrow 2} = |\mathbf{S}_{11}(00)|^2. \quad (11)$$

Radiative losses into the first and second media are

$$S_{1 \rightarrow 1} = \sum_{\alpha}^{N_1} |\mathbf{S}_{22}(\alpha 0)|^2, \quad S_{1 \rightarrow 2} = \sum_{\beta}^{N_2} |\mathbf{S}_{12}(\beta 0)|^2, \quad (12)$$

$$S_{2 \rightarrow 2} = \sum_{\alpha}^{N_1} |\mathbf{S}_{11}(\alpha 0)|^2, \quad S_{2 \rightarrow 1} = \sum_{\beta}^{N_2} |\mathbf{S}_{21}(\beta 0)|^2, \quad (13)$$

respectively. Here,  $N_i$  is a number of propagating plane waves (waves with real  $x$  component of the wave vector) in the  $i$ th medium. Conservation of energy requires

$$T_{1 \rightarrow 2} + R_{1 \rightarrow 1} + S_{1 \rightarrow 1} + S_{1 \rightarrow 2} = 1, \quad (14)$$

since the absorption is omitted. In this way we calculate transmission parameters of the surface wave propagating through the single planar interface. For more complicated structures which contain two or more interfaces, we apply the transfer matrix method. To every interface we assign its transfer matrix defined as

$$\mathbf{T}_{BA} = \begin{pmatrix} S_{12} - S_{11}S_{21}^{-1}S_{22} & S_{11}S_{21}^{-1} \\ -S_{21}^{-1}S_{22} & S_{21}^{-1} \end{pmatrix}. \quad (15)$$

The transfer matrix for the homogeneous dielectric spacing between two interfaces is

$$\mathbf{\Lambda} = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda^{-1} \end{pmatrix}. \quad (16)$$

Here  $\lambda$  is a diagonal matrix containing phase factor  $e^{ik_{x0}L}$ , where  $L$  is the width of the dielectric slab. The transfer matrix of the entire structure containing  $n$  interfaces is given as a product of successive transfer matrices

$$\mathcal{T} = \mathbf{T}_{nn-1}\mathbf{\Lambda}_{n-1}\mathbf{T}_{n-1n-2}\dots\mathbf{\Lambda}_2\mathbf{T}_{21}\mathbf{\Lambda}_1\mathbf{T}_{10}. \quad (17)$$

In the process of multiplication we have to keep in mind that some of the elements of the matrices  $\lambda_i^{-1}$  correspond to waves that increases exponentially along the  $+x$  direction. Therefore the elements of  $\mathcal{T}$  can become very large. To avoid numerical operations with  $\lambda^{-1}$  we apply the normalization procedure [12]: consider the product  $\mathcal{T} = \mathbf{T}'\mathbf{\Lambda}\mathbf{T}$ . To calculate the transmission coefficient, we need the matrix  $\mathcal{T}_{22} = S_{21}^{-1}$  which can be expressed as

$$\mathcal{T}_{22} = (0 \ 1) \begin{pmatrix} \mathcal{T}_{11} & \mathcal{T}_{12} \\ \mathcal{T}_{21} & \mathcal{T}_{22} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \quad (18)$$

This expression can be written in the form

$$\mathcal{T}_{22} = (\mathbf{T}'_{21} \ \mathbf{T}'_{22}) \begin{pmatrix} \lambda \mathbf{T}_{12} \mathbf{T}_{22}^{-1} \lambda \\ 1 \end{pmatrix} \lambda^{-1} \mathbf{T}_{22} \quad (19)$$

so that

$$S_{21} = \mathcal{T}_{22}^{-1} = T_{22}^{-1} \lambda (\mathbf{T}'_{21} \lambda \mathbf{T}_{12} \mathbf{T}_{22}^{-1} \lambda + \mathbf{T}'_{22})^{-1}, \quad (20)$$

and the transmission coefficient of the SPP  $T = |\mathbf{S}_{21}(0, 0)|^2$  is expressed without any use of elements of  $\lambda^{-1}$ . Similarly, we express matrices  $\mathcal{T}_{12}$  and  $S_{11} = \mathcal{T}_{12}S_{21}$ .

$$S_{11} = \begin{pmatrix} \mathbf{T}'_{11} \lambda \mathbf{T}_{12} \mathbf{T}_{22}^{-1} \lambda + \mathbf{T}'_{12} \\ \times (\mathbf{T}'_{21} \lambda \mathbf{T}_{12} \mathbf{T}_{22}^{-1} \lambda + \mathbf{T}'_{22})^{-1} \end{pmatrix} \quad (21)$$

which determines the reflection coefficient  $R$ . This approach can be applied recursively to the multilayer structures. We calculate  $S_{11}^{(n+1)}$  and  $S_{21}^{(n+1)}$  matrix elements for  $(n+1)$ -th interface by

$$\begin{aligned} S_{21}^{(n+1)} &= S_{21}^{(n)} \lambda Y^{-1} \\ S_{11}^{(n+1)} &= \lambda X Y^{-1} \lambda, \end{aligned} \quad (22)$$

where

$$X = T_{11}S_{11}^{(n)} + T_{12}, \quad Y = T_{21}S_{11}^{(n)} + T_{22}, \quad (23)$$

and  $S_{11}^{(n)}, S_{21}^{(n)}$  are scattering matrices for structure up to  $n$ -th interface.

#### 4. Gradient structures

Previous scattering analysis [5, 6, 7, 8] showed that the propagation of the SPP through the single dielectric interface with high index contrast is accompanied by huge scattering losses. To suppress these losses we analyze layered structure which consists of  $N$  parallel layers of the same width. The permittivity of the  $i$ -th layer is

$$\varepsilon_i = \varepsilon_A + \Delta\varepsilon \times i, \quad (24)$$

and the permittivity step  $\Delta\varepsilon = (\varepsilon_B - \varepsilon_A)/N$ . This model mimics the gradient structure where the permittivity increases linearly from  $\varepsilon_A$  to  $\varepsilon_B$ .

Figure 1 shows the transmission and the reflection coefficients and an amount of scattering losses for the normal incidence SPP propagating through such structure for  $\varepsilon_A = 1$  to  $\varepsilon_B = 11.5$ . The structure is composed from  $N = 20$  dielectric layers of the same width. The transmission increases when the width of layers increases and saturates for sufficiently wide dielectric slabs. This effect is more significant for finer permittivity steps (Fig. 2). It is obvious that radiative losses could be totally eliminated for perfectly smooth permittivity profile. We also see that the reflectivity of this structure is very small (Fig. 1).

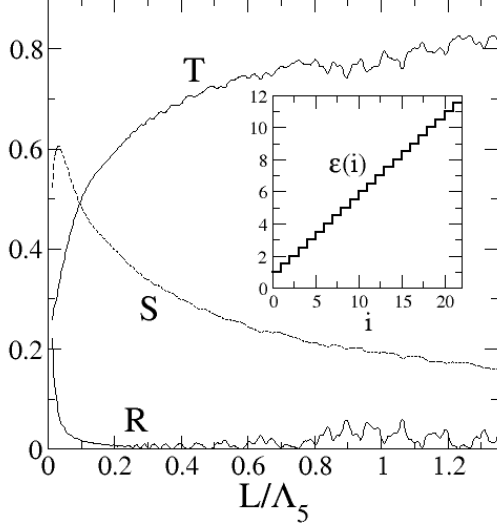


Figure 1: The transmission coefficient  $T$ , the reflection coefficient  $R$  and radiative losses  $S$  for the normal incident the SPP propagating through the gradient structure with linearly increasing value of the permittivity. The permittivity changes from  $\epsilon_A = 1$  to  $\epsilon_B = 11.5$  with the increments  $\Delta\epsilon = 0.5$  (spatial dependence of the permittivity is shown in the inset). The width  $L$  of each layer is measured relative to the wavelength  $\Lambda_5$  of the SPP. The transmission and scattering losses for the single interface  $\epsilon_A/\epsilon_B$  are  $T = 0.19$  and  $S = 0.5$ , respectively.

Of course, the limit of unity transmission coefficient can be reached only theoretically. With 20 permittivity steps, considered in Fig. 1, the entire structure becomes macroscopically large. when the width of layers is  $L = \Lambda_5 \sim 245$  nm. However, as shown in Fig. 2, considerable increase of the transmission coefficient can be obtained already with much thinner layers and/or with the use of only a few dielectric layers.

## 5. Propagation of the SPP through dielectric slab

Consider now the slab of dielectric material of width  $L_{\text{slab}}$  with permittivity  $\epsilon_B$  embedded into two semi-infinite layers with permittivity  $\epsilon_A$  and study the propagation of the SPP across such structure. The transfer matrix (17) reduces to  $\mathcal{T} = \mathbf{T}_{AB}\mathbf{\Lambda}_B\mathbf{T}_{BA}$ .

Figure 3 shows the transmission of SPP through the slab with  $\epsilon_A = 8$  and  $\epsilon_B = 1$ . The transmission and reflection coefficients exhibit typical Fabry-Perot resonances which confirm that the transmission is determined mostly by the ratio  $L_{\text{slab}}/\Lambda_1$  of the slab width

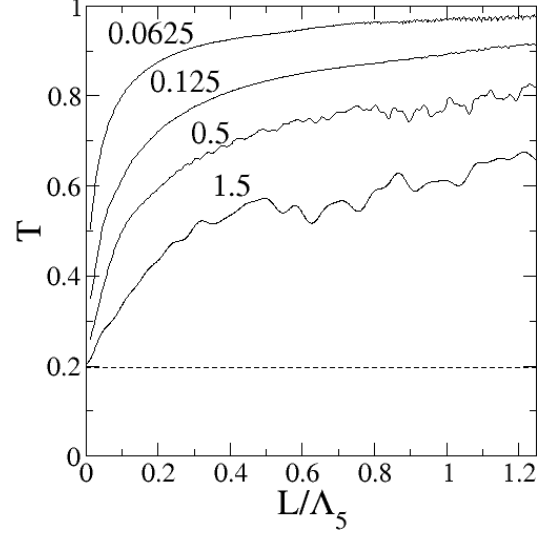


Figure 2: The transmission coefficient  $T$  for the normal incidence of the SPP propagating through the layered structure  $\epsilon_i = \epsilon_A + \Delta\epsilon \times i$  for various permittivity increments  $\Delta\epsilon$ . Horizontal dashed line represents the transmission coefficient  $T = 0.19$  for the single interface between media  $\epsilon_A = 1$  and  $\epsilon_B = 11.5$ .

to the wave length of the SPP. Similarly as in the case of the plane waves [14, 15] with maximal values for  $L = n \times \Lambda_1/2$  and maxima of the reflection  $R$  appear when  $L \approx (\Lambda_1/4) \times (2n + 1)$ . As shown in Fig. 3, the maxima of the transmission coefficients are accompanied with maxima of the scattering losses.

The width dependence of the transmission parameters for the propagation of the SPP through the layer with higher permittivity,  $B \rightarrow A \rightarrow B$ , is shown in Fig. 4. Here, the transmission coefficient exhibits dramatic oscillations when the slab thickness varies.

The difference between the two cases lies in the role of the plane waves. While in the first structure only a few plane waves are allowed to propagate through the slab (most of plane waves inside the slab are evanescent), the propagation of the SPP through the slab with higher permittivity is accompanied by a huge number of plane waves. The transmission coefficient of the plane wave  $\alpha$  is determined by the product  $Lk_{x\alpha}$ . The radiative losses are enhanced each time when the  $x$  component of the wave vector of one of the plane waves fulfills the Fabry-Perot condition for the maximal transmission. With many plane waves with different value of  $k_x$  inside the slab we observe huge oscillation of both the trans-

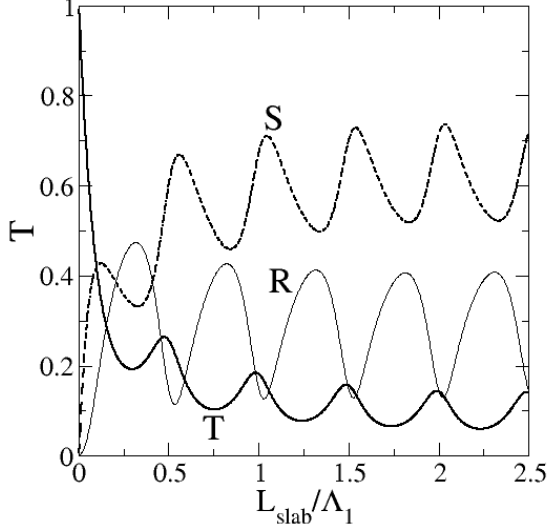


Figure 3: The transmission parameters of the normal incidence of the SPP propagating through the dielectric slab with the permittivity  $\epsilon_B = 1$  sandwiched between two dielectric media with the permittivity  $\epsilon_A = 8$ . The thickness of the slab is shown relative to wavelength of the SPP  $\Lambda_1$ . Fabry-Perot resonances of both the transmission and the reflection are clearly visible. Note that maxima of the transmission coefficient are accompanied with maxima of the radiative losses.

mission coefficient of the SPP and of radiative losses  $S$  (not shown in Fig. 4). Smooth  $L_{\text{slab}}$  - dependence of the transmission coefficient is observed only for very thin slabs<sup>1</sup>.

## 6. Quadratic structures

The propagation of the SPP through the slab, discussed in the previous section, can be enhanced with the use of gradient structures. As an example, we consider layered structure consisting of  $N$  dielectric layers with permittivities

$$\epsilon_i = 1 + (2.75 - i/4)^2, \quad i = 1 \dots N = 22. \quad (25)$$

The permittivity varies from the maximal value 8.56 to minimal value 1 in the middle of the structure (inset of Fig. 5). The width  $L$  of each slab is again considered relative to the length of the SPP for the dielectric permittivity  $\epsilon_d = 1$  ( $\Lambda_1 = 632 \text{ nm}$ ).

<sup>1</sup> Note that the wave length  $\Lambda_8$  of the SPP is considerably smaller than  $2\pi/k_x$  for any accessible plane wave.

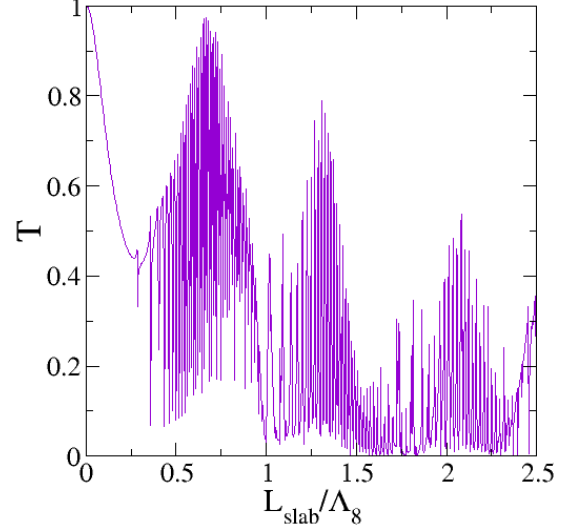


Figure 4: The transmission coefficient  $T$ , the reflection coefficient  $R$  and scattering losses for the propagation of the SPP through the dielectric slab with the permittivity  $\epsilon_B = 8$  sandwiched between dielectric media with the permittivities  $\epsilon_A = 1$ . The thickness of the is shown relative to wavelength of the SPP  $\Lambda_8$ .

The transmission parameters of the SPP are shown in Fig. 5. Contrary to the propagation through the slab (Fig. 3), no Fabry-Perot oscillations are observable. Instead, we see that the transmission coefficient increases continuously and the reflection coefficient is almost negligible.

The inverse structure, defined by the quadratic increase of permittivity

$$\epsilon_i = 8.5625 - (2.75 - i/8)^2 \quad i = 0 \dots 44, \quad (26)$$

exhibits oscillations of the transmission coefficient, which are, however, much more moderate than that shown in Fig. 4 for a single slab. The number of oscillations is proportional to a number of permittivity steps. The transmission coefficient and radiation losses are anti-correlated: small radiation losses correspond to large transmission coefficient and *vice versa*.

## 7. Propagation of the SPP through periodic structures

In this section we consider the propagation of the SPP through the layered structure built by  $N$  periods of layers  $ABAB \dots AB$  with permittivities  $\epsilon_A$  and  $\epsilon_B$ . The pe-

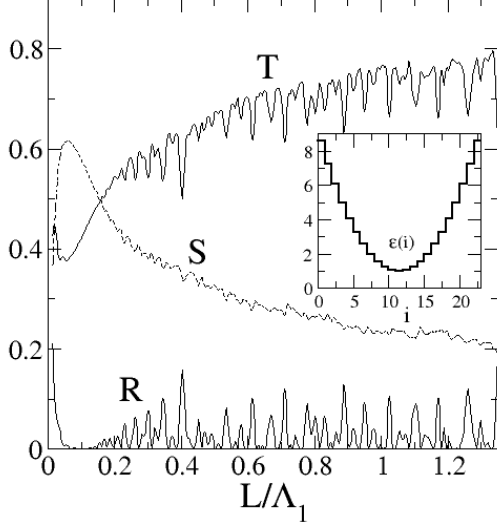


Figure 5: The transmission coefficient  $T$ , the reflection coefficient  $R$  and radiative losses  $S$  for the normal incidence SPP propagating through the layered structure with quadratic change of the dielectric permittivity. Permittivity  $\epsilon_d = 1 + (2.75 - i/4)^2$  for  $i = 0 \dots 22$  is shown in the Inset. The width of all layers is same and is shown relative to  $\Lambda_1$ .

riodic structure can be either sandwiched between two semi-infinite media  $A$  or between medium  $A$  and  $B$  from the left (right) hand side.

We find that the SPP propagating through such structure can radiate less energy than the process of the propagation through a single interface  $AB$ . Also, radiation losses do not increase as the number of stack increases. Waves radiated at the first interface are used in the process of scattering at next interface.

Typical transmission properties of the periodic stack are shown in Figs. 7 and 8. The structure consists of  $N = 30$  periods  $AB$ . The width of each layer is proportional to the wavelength of the SPP:

$$\frac{L_A}{L_B} = \frac{\Lambda_A}{\Lambda_B}. \quad (27)$$

and permittivities  $\epsilon_A = 1$  and  $\epsilon_B = 2$ .

Figure 7 presents the transmission parameters for the propagation of the SPP through  $N = 30$  periods  $AB$ . Both left and right medium have the permittivity  $\epsilon_A = 1$ . Transmission and reflection coefficients exhibit typical oscillations when the layer widths increase. For  $L_A/\Lambda_A > 0.22$ , we found the gap in which no propagation of the SPP is possible. Similarly to plane waves, the

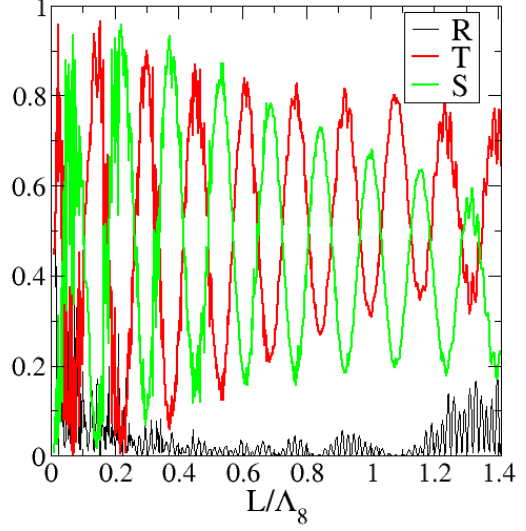


Figure 6: The transmission coefficient  $T$ , the reflection coefficient  $R$  and scattering losses  $S$  for the normal incidence SPP propagating through the layered structure with quadratic change of the dielectric permittivity  $\epsilon_d = 8.5625 - (2.75 - i/8)^2$  for  $i = 0 \dots 44$ . Width of all layers is same and is shown relative to  $\Lambda_8$ .

origin of the transmission gap lies in the wave character of the SPP. However, there are two differences between the gap for SPP and for plane waves in one dimensional photonic crystals. First, the reflection coefficient of the SPP does not reach the unity inside the gap. A small radiative losses are still observed since there is no total gap for the TM polarized plane waves [16]. Second, since plane waves must assist the propagation of the SPP through each interface inside the layered structure, we expect that the transmission coefficient of the SPP is considerably reduced for layers widths where the propagation of majority of plane waves is restricted. Also, the position of the gap for the SPP does not scale with the frequency because of the non-linear frequency dependence of the wave length  $\Lambda$ .

Figure 8 shows similar data for the propagation through  $N = 30$  periods  $AB$  but with right medium  $\epsilon_B = 2$ . The radiation losses are small for any width of layers. Plane waves radiated at the first interface into the material  $B$  assist the transmission of the SPP through next interfaces  $BA$  so no additional waves should be radiated. Also, for thin layers, the transmission coefficient is higher than the transmission coefficient for the single interface  $AB$  ( $T_0 = 0.85$ ).

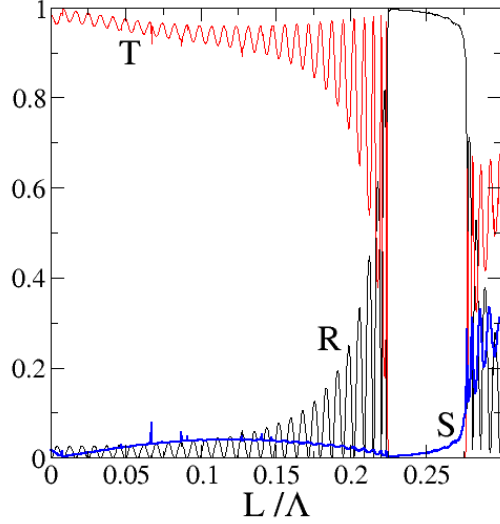


Figure 7: The transmission coefficient  $T$ , the reflection coefficient  $R$  and scattering losses  $S$  for the normal incidence of the SPP propagating through the periodic structure of two alternating media as a function of layer thickness. Layers are arranged in order  $ABAB \dots ABA$ , number of periods is  $N = 30$ . Permittivities are  $\epsilon_A = 1$  and  $\epsilon_B = 2$ ,  $N = 30$ . Widths of layers are given by Eq. (27).

The last two Figures, 9 and 10 demonstrate the transmission parameters for the SPP propagating through periodic layered structure with single impurity represented by additional layer  $B$  or  $A$  in the middle of the structure. Similarly to the plane wave photonic crystals, impurity creates isolated level in the band gap, where the reflection coefficient decreases. Figure 9 shows that this decrease of the reflection should not automatically imply a good transmission of the SPP; instead, decrease of the reflection is accompanied with plane wave radiation and the transmission coefficient is small. The transmission of SPP is better for the  $A$  impurity layer included into the layered media with slower permittivity contrast. For instance, Fig. 10 shows well pronounced impurity induced transmission peak ( $T \approx 0.7$  inside the gap for  $\epsilon_A/\epsilon_B = 3/5$  with impurity  $A$  located in the middle of the periodic structure).

## 8. Conclusion

We analyzed the propagation of the surface plasmon polariton - SPP - through various layered structures in which the metallic surface is covered by strips of dielec-

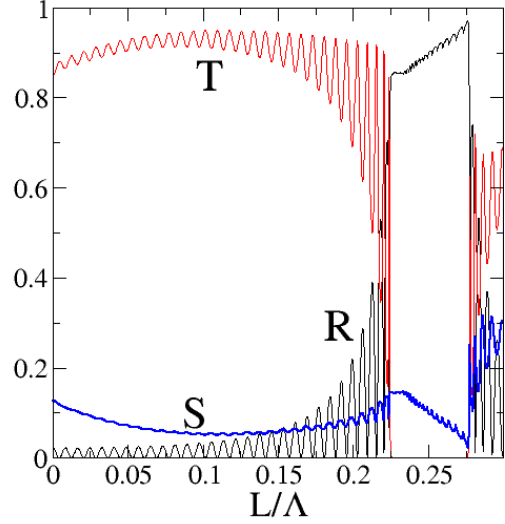


Figure 8: The transmission coefficient  $T$ , the reflection coefficient  $R$  and scattering losses  $S$  for the normal incidence of the SPP propagating through the periodic structure of two alternating media as a function of layer thickness. Layers are arranged in order  $ABAB \dots AB$ , number of periods is  $N = 30$ . Permittivities are  $\epsilon_A = 1$  and  $\epsilon_B = 2$ . Widths of layers are given by Eq. (27). Note that the transmission through such structure can be larger than the transmission through single interface,  $T_0 = 0.86$ .

tric materials with different dielectric permittivity. We showed that the transmission coefficient of SPP can be considerably increased when the single permittivity step with high permittivity contrast is substituted by a gradient structure, represented by  $N$  thin dielectric slabs with increasing dielectric permittivity.

We showed that the propagation of the SPP through more than one dielectric interface must not necessary require an increase of the radiative losses. Plane waves radiated in one interface assist in the process of the transmission of the SPP through the next interface.

Although no absorption is considered in our analysis, we expect that qualitatively the same results in absorbing structures. Absorption only reduces the transmission coefficient for the SPP and radiative losses.

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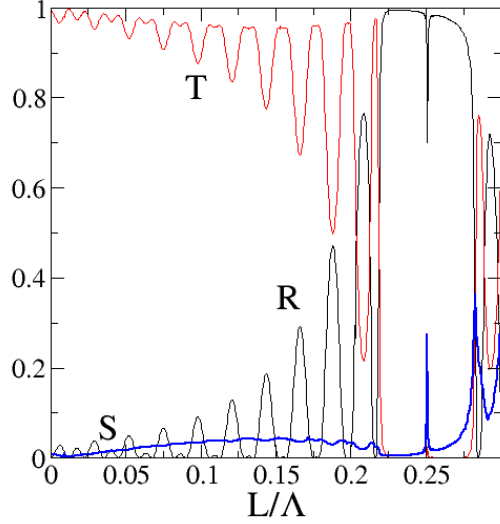


Figure 9: The transmission coefficient  $T$ , the reflection coefficient  $R$  and scattering losses  $S$  for the normal incidence of the SPP propagating through the periodic structure of two alternating media with additional layer  $B$  located in the middle of the structure as a function of layer thickness. Layers are arranged in order  $ABAB \dots ABBA \dots ABA$ , number of periods is  $N = 22$ . Permittivities are  $\epsilon_A = 1$  and  $\epsilon_B = 2$ .

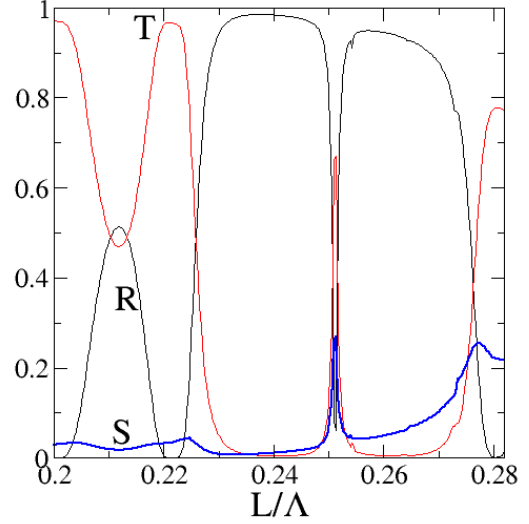


Figure 10: The transmission coefficient  $T$ , the reflection coefficient  $R$  and scattering losses  $S$  for the normal incidence of the SPP propagating through the periodic structure of two alternating media with additional layer  $A$  located in the middle of the structure as a function of layer thickness. Layers are arranged in order  $ABAB \dots BAAB \dots ABA$ , number of periods is  $N = 22$ . Permittivities are  $\epsilon_A = 3$  and  $\epsilon_B = 5$ .

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